

Mark Scheme (Result)

October 2020

Pearson Edexcel GCE In A level Further Mathematics Paper 9FM0/4D

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at <u>www.edexcel.com</u> or <u>www.btec.co.uk</u>. Alternatively, you can get in touch with us using the details on our contact us page at <u>www.edexcel.com/contactus</u>.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Ocotber 2020 Publications Code 9FM0_4D_2010_MS All the material in this publication is copyright © Pearson Education Ltd 2020

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 75.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: method marks are awarded for `knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.
- 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Question	Scheme	Marks	AOs
1(a)	Subtracting each entry from a constant value ≥ 35 to convert from maximisation problem to minimisation	M1	1.1b
	Add a sufficiently large number (> 15) to cells A3 and B4 e.g. $\begin{bmatrix} 6 & 15 & 100 & 12 \\ 3 & 5 & 7 & 100 \\ 0 & 3 & 1 & 10 \\ 6 & 4 & 8 & 5 \end{bmatrix}$	B1	1.1b
	Reduce rows $\begin{bmatrix} 0 & 9 & 94 & 6 \\ 0 & 2 & 4 & 97 \\ 0 & 3 & 1 & 10 \\ 2 & 0 & 4 & 1 \end{bmatrix}$ and then columns $\begin{bmatrix} 0 & 9 & 93 & 5 \\ 0 & 2 & 3 & 96 \\ 0 & 3 & 0 & 9 \\ 2 & 0 & 3 & 0 \end{bmatrix}$	M1 A1ft	2.1 1.1b
	followed by $\begin{bmatrix} 0 & 7 & 93 & 3 \\ 0 & 0 & 3 & 94 \\ 0 & 1 & 0 & 7 \\ 4 & 0 & 5 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 7 & 91 & 3 \\ 0 & 0 & 1 & 94 \\ 2 & 3 & 0 & 9 \\ 4 & 0 & 3 & 0 \end{bmatrix}$	M1 A1ft	2.1 1.1b
	A – 1, B – 2, C – 3, D – 4	B1ft	1.1b
		(7)	
(b)	£123	B1	1.1b
		(1)	
	1	(8 n	narks)
Notes:			
B1: addi M1: sim A1ft: ca M1: dev one A1ft: ca (b) B1ft: co (a)	wert from maximisation to minimisation (allow at most two errors) ing a large number (at least 16) to cells A3 and B4 aplifying the initial matrix by reducing rows and then columns to following on from their earlier subtraction welop an improved solution – need to see one double covered +e; one unc e single covered unchanged. 3 lines needed to 4 lines needed to following on from row and column reduction final table prrect allocation ft their optimal table (all previous M marks must have be of one column of original problem		

Question	Scheme	Marks	AOs
2 (a)	EMV for A is $0.6(350) + 0.4(-140) = 154$		
	EMV for B is $0.75(260) + 0.25(-190) = 147.5$	M1 A1	3.4 1.1b
	EMV for C is $0.8(220) + 0.2(-230) = 130$		1.10
	The optimal EMV is \pounds 154, which makes option A the best choice using the EMV criterion	A1	2.2a
		(3)	
(b)	u(350) = 0.5831379803, u(-140) = -0.4190675486		
	u(260) = 0.4779542232, u(-190) = -0.6080141975	M1	3.4
	u(220) = 0.4230501896, u(-230) = -0.7771305269		
	Calculate all three expected utilities:		
	A is $0.6(0.583) + 0.4(-0.419) = 0.1822557$	DM1	1.1b
	B is $0.75(0.477) + 0.25(-0.608) = 0.2064621$	A1	1.1b
	C is $0.8(0.423) + 0.2(-0.777) = 0.1830140$		
	The optimal expected utility is 0.206 utils, which makes option B the best choice using expected utility as the criterion	A1	2.2a
		(4)	
	1	(7 n	narks)
Notes:			
(a) M1: Co	rrect method for calculation EMV for either A, B or C		
	rrect values of EMV for A, B and C		
A1: Cor	rrect deduction of optimal EMV (dependent on all three correct EMVs)		

(b) M1: Uses the correct utility function to replace each pay-off with the corresponding utility DM1: Calculate all three expected utilities using correct probability values from (a) A1: At least 2 expected utilities correct (correct to at least 2 decimal places) A1: Correct deduction of optimal expected utility

Question	Scheme	Marks	AOs
3 (a)	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	M1 A1	2.1 1.1b
		(2)	
(b)	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	M1 A1	1.1b 1.1b
	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	M1	1.1b
	Entering cell is DQ and exiting cell is AP	A1	2.2a
		(4)	
(c)	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	M1 A1	2.1 1.1b
	No negative IIs so solution is optimal	A1	2.4
		(3)	
(d)	Let x_{ij} be the number of units (of stock) transported from (supply point) <i>i</i> to (sales point) <i>j</i>	B1	3.3
	where $i \in \{A, B, C, D\}$ and $j \in \{P, Q, R\} (x_{ij} \ge 0)$	B1	3.3
	Minimise $25x_{AP} + 24x_{AQ} + 17x_{AR} + 7x_{BP} + 12x_{BQ} + 14x_{BR}$ + $13x_{CP} + 11x_{CQ} + 20x_{CR} + 16x_{DP} + 15x_{DQ} + 13x_{DR}$	B1 B1	2.5 3.3

	$\sum x_{A_j} \le 42, \sum x_{B_j} \le 68, \sum x_{C_j} \le 25, \sum x_{D_j} \le 40 $ accept =	B1	3.3
	$\sum x_{iP} \ge 59, \sum x_{iQ} \ge 72, \sum x_{iR} \ge 44 \qquad \text{accept} =$	B1	3.3
		(6)	
	The Simplex algorithm cannot be used as not all the constraints are in	1	
(e)	the form $\sum x \le k$ where k is a positive constant	B1	3.5b
		(1)	
		(16	marks)
Notes:			
(c) M1: fr p la	AO – including the deduction of all entering and exiting cells nding all 7 shadow costs and the 6 improvement indices – this mark is d revious M mark in (b) which will therefore indicate a correct mathematic		
A1: C A1: C	eading from the initial solution to the confirmation or not of the optimality obtain AO (shadow costs and IIs) SO including the correct reasoning that the solution is optimal because the egative II porrect definition of x_{i}		
A1: C A1: C (d) B1: C B1: C B1: C B1: C B1: C	olution AO (shadow costs and IIs) SO including the correct reasoning that the solution is optimal because th	nere are no	rrent

Question	Scheme	Marks	AOs			
4 (a)	CF of $u_n = A(2)_n + B(-1)_n \Rightarrow$ auximary equation is					
	(m-2)(m+1) = 0	M1	3.1a			
	$m^2 - m - 2 = 0 \Longrightarrow \alpha = -1, \beta = -2$	A1	2.2a			
		(2)				
	particular solution if $u_n = \lambda (-s)_n$, $u = \lambda (-s)_{n+1}$, $u = \lambda (-s)_{n+2}$					
(b)	<i>n</i> +1 <i>n</i> +2	M1	2.1			
	$9\lambda + 3\lambda - 2\lambda = 20 \ (\Rightarrow \lambda = 2)$	M1	1.1b			
	$u_n = A(2)_n + B(-1)_n + 2(-3)_n$	A1	1.1b			
	$2u_0 = u_1 \Longrightarrow 2A + 2B + 4 = 2A - B - 6$	M1	1.1b			
	$u_4 = 164 \Longrightarrow 16A + B + 162 = 164$	M1	1.1b			
	$A = \frac{1}{3}, B = -\frac{10}{3} \Longrightarrow u_n = \frac{1}{3} (2)^n - \frac{10}{3} (-1)^n + 2 (-3)^n$	A1	2.2a			
		(6)				
		(8 n	narks)			
Notes:						
(a) M1: Use	es given complementary function to find auxiliary equation correspondin	g to secon	d-			
ord	er recurrence relation					
A1: cao (b) M1: sub	A1: cao for both α and β b) M1: substitute $u_n = \lambda (-3)_n$ into their second-order recurrence relation					
M1: for	M1: forms linear equation in λ only					
A1: corr	A1: correct general solution					
M1: use $2u_0 = u_1$ to form an equation in <i>B</i> (and possibly <i>A</i>)						
M1: use $u_4 = 164$ to set up a second equation in <i>A</i> and <i>B</i>						
A1: cao						

Question	Scheme	Marks	AOs
5 (a)	Source node is C	B1	1.1b
		(1)	
(b)	G is the sink node as all the arcs incident to G flow into G	B1	2.4
		(1)	
(c)	Capacity of cut $C_1 = 10 + 2 - 1 + 6 + 8 + 1 - 0 = 26$	B1	1.1b
		(1)	
(d)(i)	Arc JH must be at its upper capacity of 5 as the two arcs that flow into J (EJ and FJ) have a lower capacity of $2 + 3 = 5$	B1	2.4
(ii)	Arcs AD and CD must be at the lower capacities (which in total is 9) as the only two arcs (DG and DE) that flow out of D have a total upper capacity of $7 + 2 = 9$	B1	2.4
		(2)	
(e)	A 5 D 7 G 10 H 10 H 6 4 4 3 5 5 5 5 5 5 5 5 5 5	M1 A1	2.2a 1.1b
		(2)	
	Use of max-flow min-cut theorem	M1	2.1
(f)	Identification of cut through DG, DE, CE, CF, CB, BA with a capacity of 18 and value of flow = 18	A1	3.1a
	Therefore it follows that flow is maximal	A1	2.2a
		(3)	
	1	(10 n	narks)
Notes:			

(a) **B1:** cao (node C)

(b) **B1:** correct explanation of why G is the sink node

(c) **B1:** cao

(d)(i) B1: correct explanation that JH must be at its upper capacity (must refer to arcs EJ and FJ) (d)(ii) B1: correct explanation that AD and CD must be at their lower capacities (must refer to arcs DG and DE)

(e) M1: 'flow in = flow out' at all but one vertex – one number only required on each arc (condone blank for arc BF)

A1: a correct valid flow through the network (check that flow in must equal flow out at each vertex)

(f) M1: Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through saturated arcs)

A1: Use appropriate process of finding a minimum cut (cut + value correct)

A1: Correct deduction that the flow is maximal

Question	Scheme	Marks	AOs
6 (a)	Option R (or option T) dominates option S	B1	1.2
	Because e.g. $4 > 2$ and $-3 > -4$ and $1 > -2$	B1	2.4
		(2)	
(b)	Row minima: 1, -3, -2 max is 1	M1	1.1t
	Column maxima: 4, 5, 3 min is 3	A1	1.18
	Row maximin (1) \neq Column minimax (3) so not stable	A1	2.4
		(3)	
(c)	V is less than or equal to each of these three expressions since we need to find the maximum value of the worst possible augmented expected pay-off for each value of p	B1	2.3
		(1)	
(d)	It is necessary to use an inequality because it enables the Simplex algorithm to pivot on a row that will increase the value of P	B1	3.5
		(1)	
(e)	$p_2 = \frac{4}{11}$	B1	1.11
	Substitute <i>p</i> values to obtain $V \le \frac{56}{11}, \frac{56}{11}, \frac{58}{11}$	M1	3.4
	Value of the game to player A = $\frac{56}{11} - 3 = \frac{23}{11}$	A1	2.2a
		(3)	
(f)	$\begin{array}{rcl} q_{1} + 5q_{2} + 3q_{3} &=& \frac{23}{11} \\ 4q_{1} - 3q_{2} + q_{3} &=& \frac{23}{11} \\ q_{1} + q_{2} + q_{3} &=& \boxed{1} \end{array}$	M1 A1ft A1	3.1a 1.1t 1.1t
	Player B should play option X with probability $\frac{8}{11}$, option Y with probability $\frac{3}{11}$ and never play option Z	A1	3.2
		(4)	
	1	(14 m	_ narks

B1: correct inequalities - must be clear that all inequalities must hold

(b) M1: attempt at row minima and column maxima – condone one error A1: correct max(row min) and min(col max)
A1: correct reasoning that the game is not stable (accept 1 ≠ 3 + statement)

(c) **B1:** an understanding that for each value of *p* we are seeking the minimum possible output

- (d) B1: as a minimum accept an answer that implies that an inequality is required so that we can apply the Simplex algorithm
- (e) **B1:** cao

M1: substitute their p values into all three expressions for the upper bound of V

A1: cao for the value of the game to player A

(f) M1: Attempt to set up at least three equations in q_1, q_2, q_3 using the value of the game from (e)

A1ft: Two correct ft "their" V

A1: cao (for exactly three equations correct)

A1: cao in context

uestion			S	Scheme			Marks	AOs
7(a)	Stage	State	Action	Dest.	Value			
	Trainers	0	0	0	0		D 1	
		1	1	0	50		B1	3.1a
		2	2	0	90			
		3	3	0	170			
		4	4	0	225			
		5	5	0	295			
	Sandals	0	0	0	0			
		1	1	0	$70 + 0 = 70^*$		M1	3.1a
			0	1	0 + 50 = 50		A1	1.11
		2	2	0	110 + 0 = 110		A1	1.11
			1	1	70 + 50 = 120*			
			0	2	0 + 90 = 90			
		3	3	0	165 + 0 = 165			
			2	1	110 + 50 = 160			
			1	2	70 + 90 = 160			
			0	3	0 + 170 = 170*			
		4	4	0	245 + 0 = 245*		M1	1.11
			3	1	165 + 50 = 215		A1	1.1
			2	2	110 + 90 = 200		A1 A1	1.1
			1	3	70 + 170 = 240		AI	1.1
			0	4	0 + 225 = 225			
		5	5	0	300 + 0 = 300*			
			4	1	245 + 50 = 295			
			3	2	165 + 90 = 255			
			2	3	110 + 170 = 280			
			1	4	70 + 225 = 295			
			0	5	0 + 295 = 295			
	High	5	5	0	305 + 0 = 305			
	heels		4	1	235 + 70 = 305		M1	1.1
			3	2	x + 120		A1ft	1.1
			2	3	115 + 170 = 285			
			1	4	75 + 245 = 320			
			0	5	0 + 300 = 300			
	320 and <i>x</i> -	+120					A1ft	1.11
							(10)	
(b)	Trainers: 0		Sandals: 4	High h	eels: 1		B 1	1.11
	Trainers: 1		Sandals: 1	High h	eels: 3		B 1	2.2
							(2)	
	1					I	(12 n	narks

Notes:
(a) B1: CAO for the first stage (all six rows) – entries in all columns must be correct – candidates
may start with state 5 (rather than state 0) which is fine
M1: Second stage – my states 1, 2 and 3 (so at least 9 rows in the first half of the second stage or
at least 20 non-zero rows). Value column must be complete with at least one value correct
for each state – ignore entries in all other columns
A1: Value column for states 1, 2 and 3 correct for the second stage – ignore entries in all other
columns and condone additional rows
A1: CAO for states 0, 1, 2 and 3 of the second stage (no additional rows for these four states) –
entries in all columns must be correct
M1: Second stage – my states 4 and 5 (so at least 11 rows in the second half of the second stage
or at least 20 non-zero rows). Value column must be complete with at least one value correct
for each state – ignore entries in all other columns
A1: Value column for states 4 and 5 correct for the second stage – ignore entries in all other
columns and condone additional rows
A1: CAO for states 4 and 5 of the second stage (no additional rows for these two states) - entries
in all columns must be correct
M1: At least 6 rows for the third stage. Value column must be complete with at least 3 values
correct – ignore entries in all other columns
A1ft: CAO for third stage correct (no additional rows for this stage) - entries in all columns must
be correct
A1ft: Must have earned all previous M marks from their completed dynamic programming but ft
their result
(b) B1: One correct allocation (dependent on first three M marks in (a))
B1: For both correct (dependent on first three M marks in (a))

Pearson Education Limited. Registered company number 872828 with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom